Space-Charge Induced Transport Limits in Periodic Quadrupole Focusing Channels*

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Overview: Machine Operating Points

Good transport of an unbunched, single component ion beam with intense space-charge requires hierarchy conditions:

C1. Lowest Order:

Stable centroid <=> single-particle orbit

C2. Next Order:

Stable 2nd order moment description <=> rms envelope equation

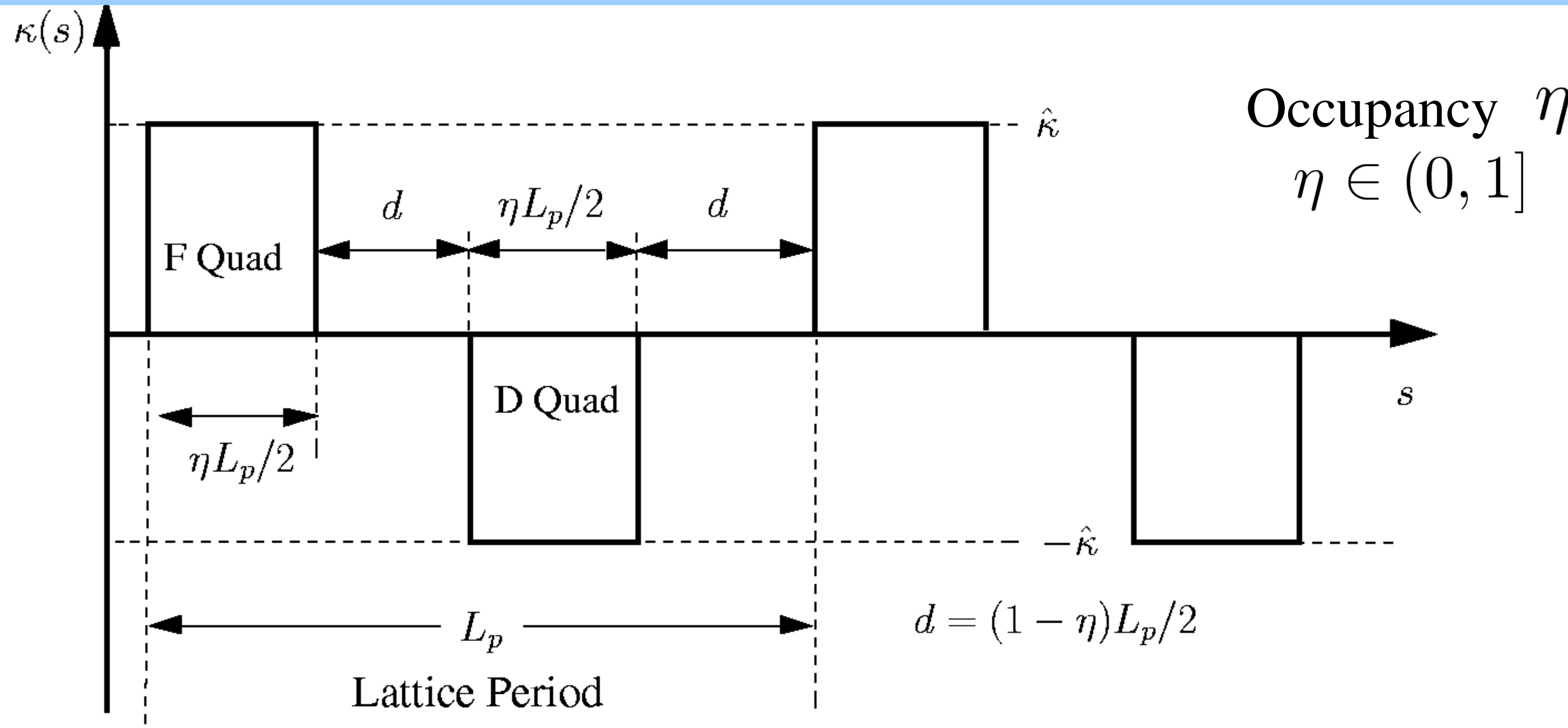
C3. Higher Order:

"Stable" within Vlasov-Poisson model

Vlasov stability practically defined with respect to limiting statistical (rms) emittance growth and particle losses for an initial distribution that is both *relatively smooth* and *plane equilibrated*. Stability realized by suppressing:

- Collective modes internal to the beam becoming unstable and growing
- Excessive halo generated
  - including nonlinear waves evolving well outside core
- Combined processes above

Stability analyzed for a coasting beam in a periodic FODO quadrupole lattice with a piecewise constant focusing function



- Stability results insensitive to fringe field structure and occupancy η

C1: First stability condition of stable centroid (single particle orbit) satisfied for any periodic lattice when:

$$\boxed{\sigma_0 < 180^\circ/\text{Lattice Period}}$$

σ_0 = single particle phase advance in the applied focusing field

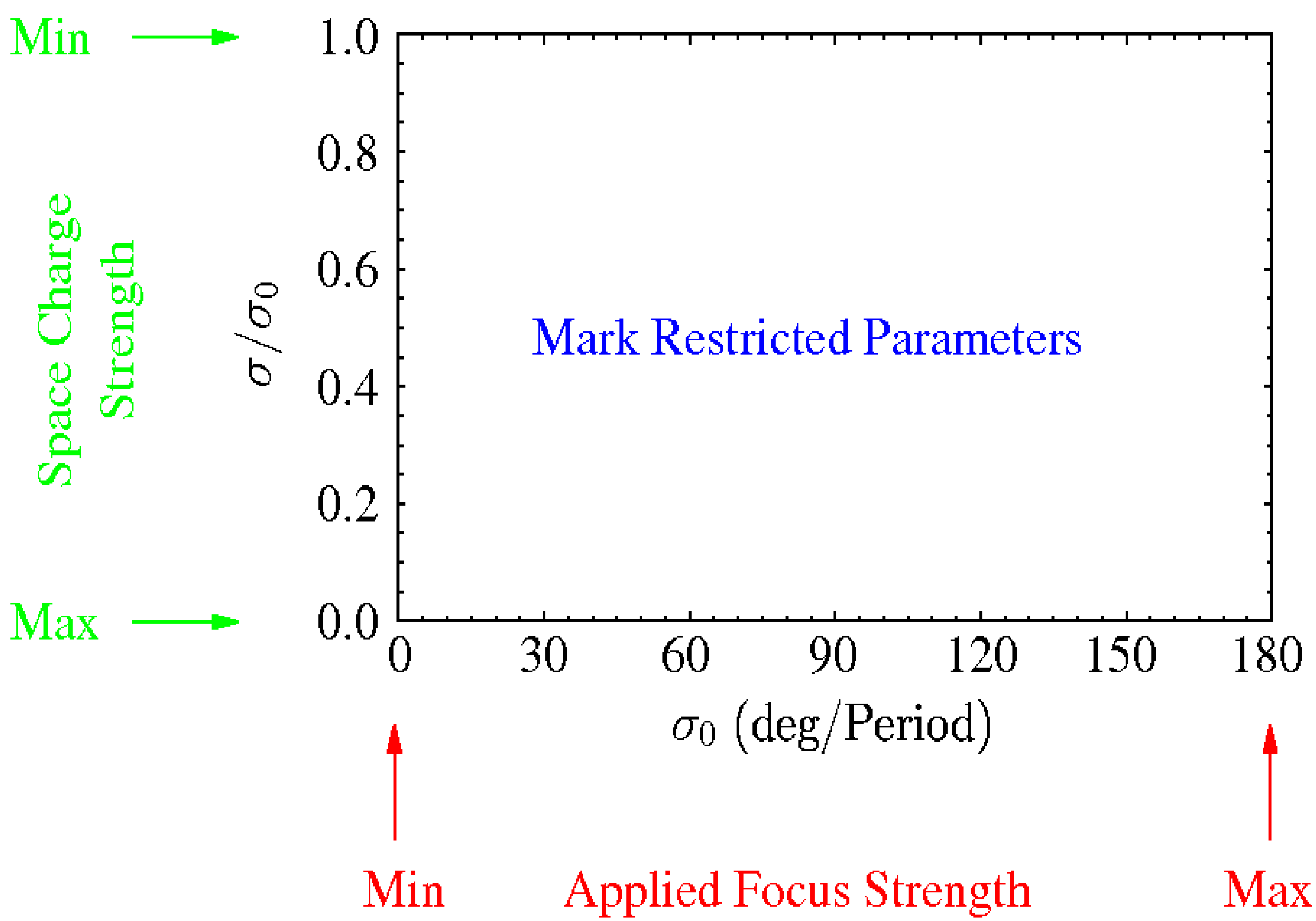
Mark regions in parameter-space where transport limits result

Use depressed and undepressed phase-advance parameters:

$$\sigma_0 = \text{particle phase advance in applied focusing field}$$

$$\sigma = \text{particle phase advance including applied focus + space-charge}$$

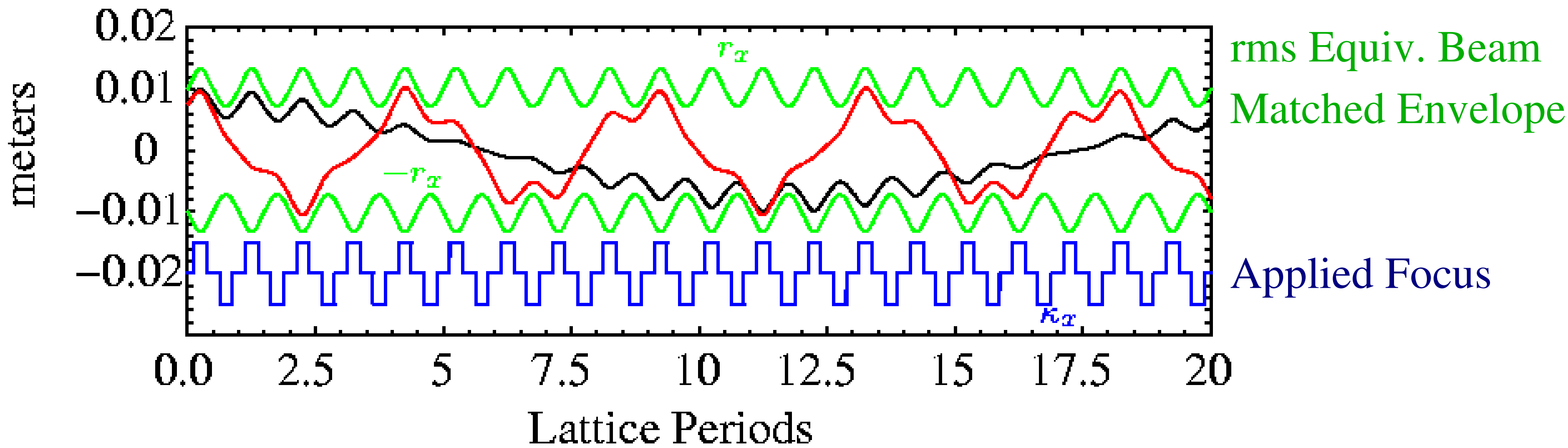
- Also other parameter (e.g. lattice period L_p and focusing occupancy η)
- Little dependence on other parameters found for *quadrupole* transport limits



To measure relative strength of space-charge we employ the ratio of particle phase-advances in presence/absence of space-charge σ/σ_0

x-plane orbits in beam $(\sigma/\sigma_0 = 0.1)$:

Black	-	Depressed	$\sigma = 9^\circ$	with rms equiv beam space-charge
Red	-	Undepressed	$\sigma_0 = 90^\circ$	applied focusing only



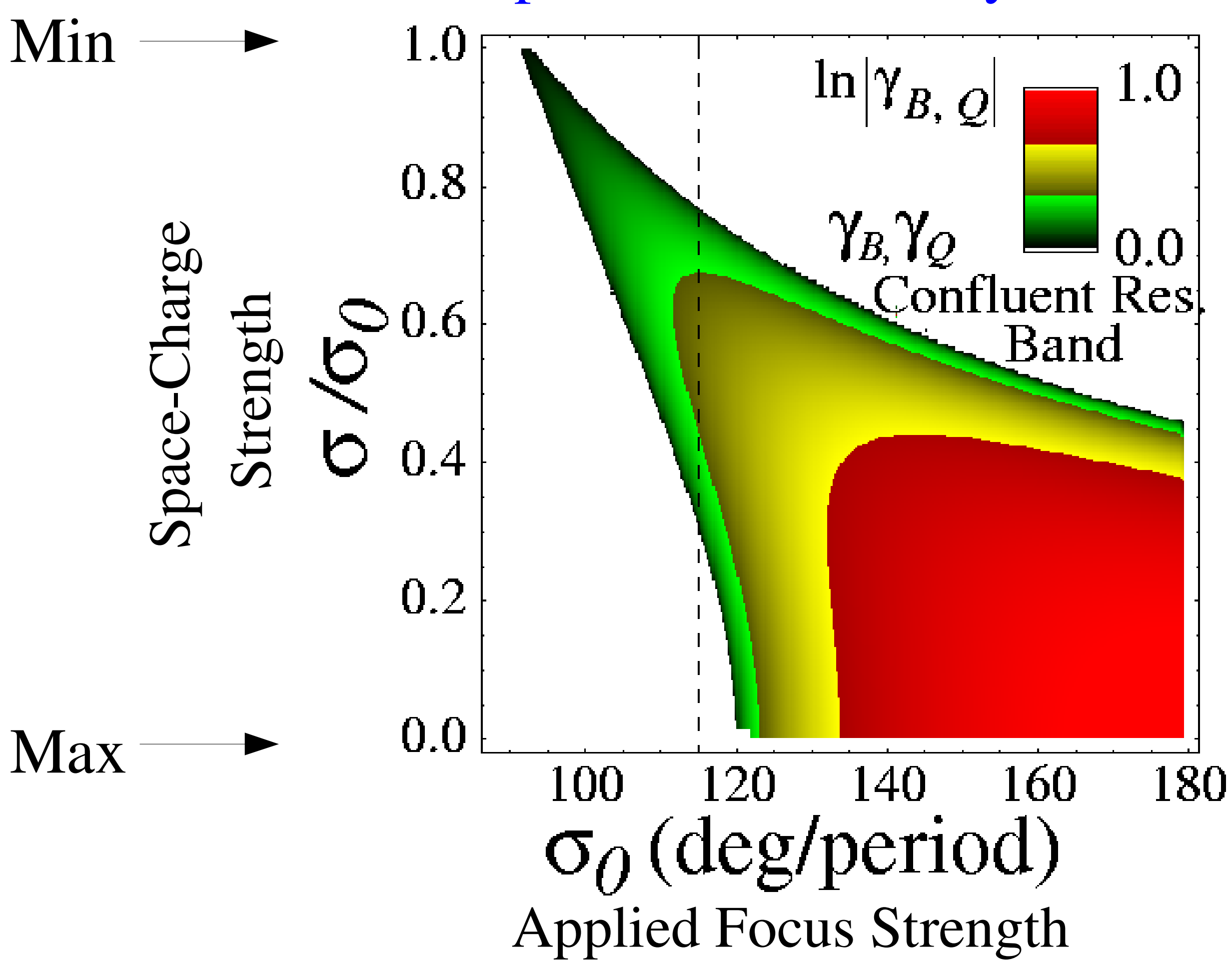
$$0 \leq \frac{\sigma}{\sigma_0} \leq 1$$

Max Space-Charge Intensity

No Space-Charge Intensity

C2: 2nd order moment instabilities described by the rms envelope equations are well understood and must be avoided for reliable quadrupole transport

Envelope Mode Instability Growth Rates



Original:

Struckmeier and Reiser, *Part. Accel.* **14**, 227 (1984)

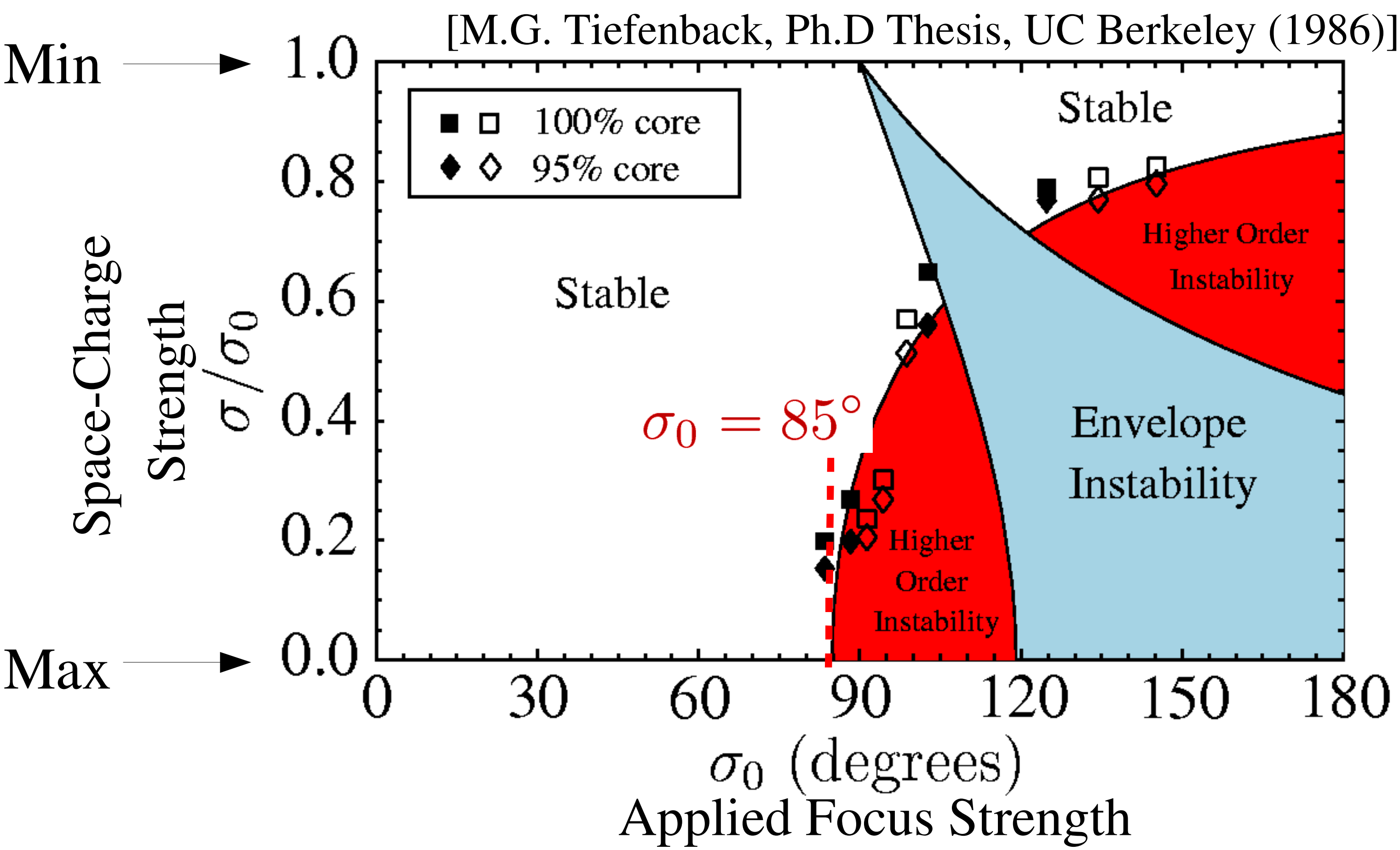
Review with full instability characterizations:

Lund, Bukh, *PRSTAB* **7**, 024801 (2004)

- rms envelope eqs + const emittances reliable predictor of moment instabilities
- For periodic quadrupole channels, “breathing” and “quadrupole” modes merge to form a “confluent” resonance band of instability

In the SBTE experiment at LBNL:

C3: Higher order Vlasov instability with strong emittance growth/particle losses observed in broad parametric region below envelope band



Results summarized by $\sigma_0 \lesssim 85°$ for strong space-charge

- Reliably applied design criterion in the lab
- Limited theory understanding for 20+ years; Haber, Laslett simulations supported

Self consistent Vlasov stability simulations were carried out to better quantify characteristics of instability

- Carried out using the WARP PIC code from LLNL/LBNL
- High resolution/stat 2D *x-y* slice simulations time-advanced to *s*-plane
- Non-singular, rms matched distributions loaded:
  - semi-Gaussian
  - Continuous focusing equilibrium $f(H)$ with self-consistent space-charge transformed to alternating-gradient symmetry:
    - waterbag
    - parabolic
    - Gaussian/Thermal

More details:

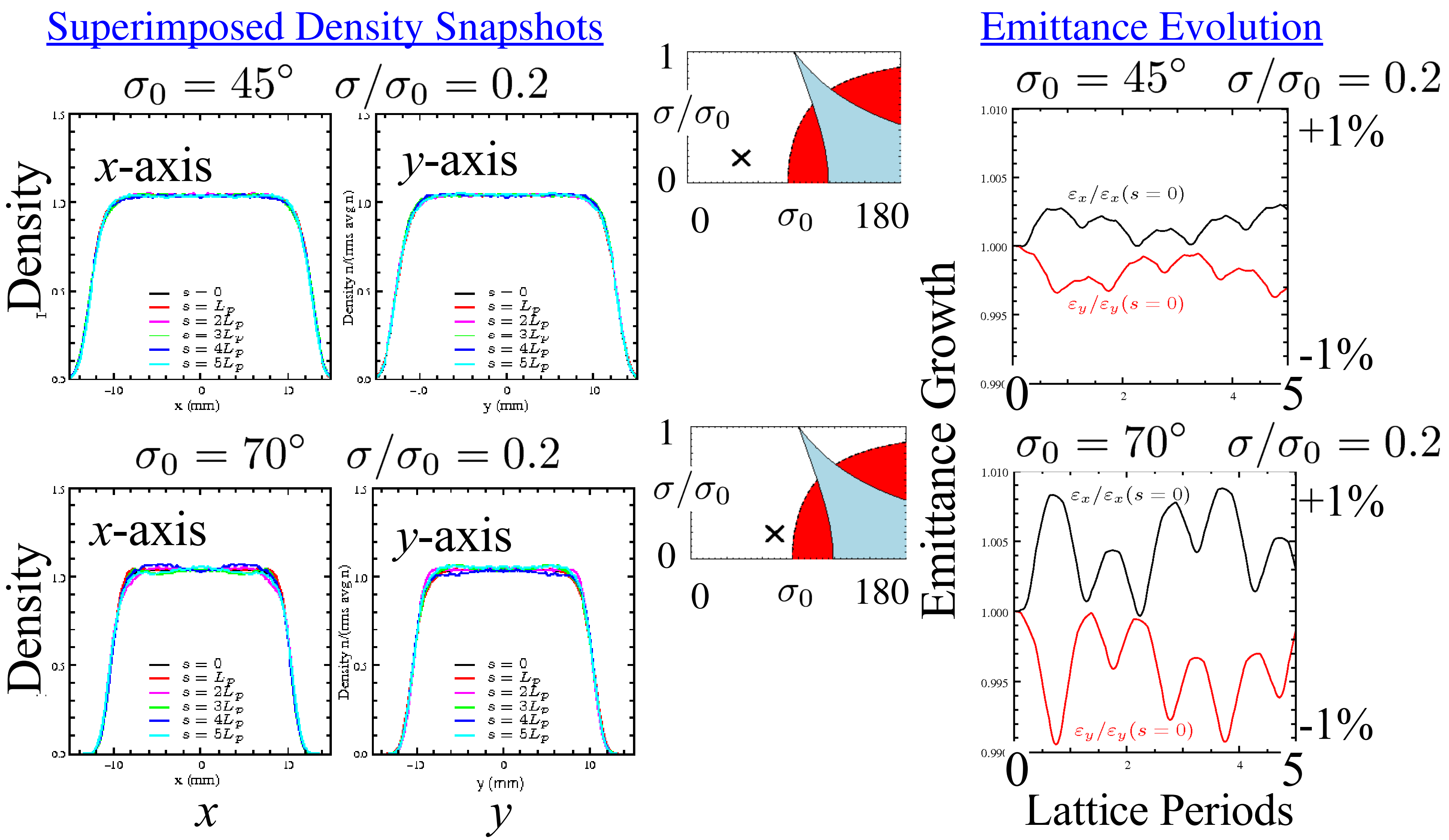
Lund, Kikuchi, Davidson, "Generation of initial distributions for simulations with high space-charge intensity," *PRSTAB* submitted

- Singular KV also loaded - only to check instability resolutions

Parametric simulations of non-singular, initially rms matched distributions have little emittance evolution outside of instability regions experimentally observed

Example: initial thermal equilibrium distribution

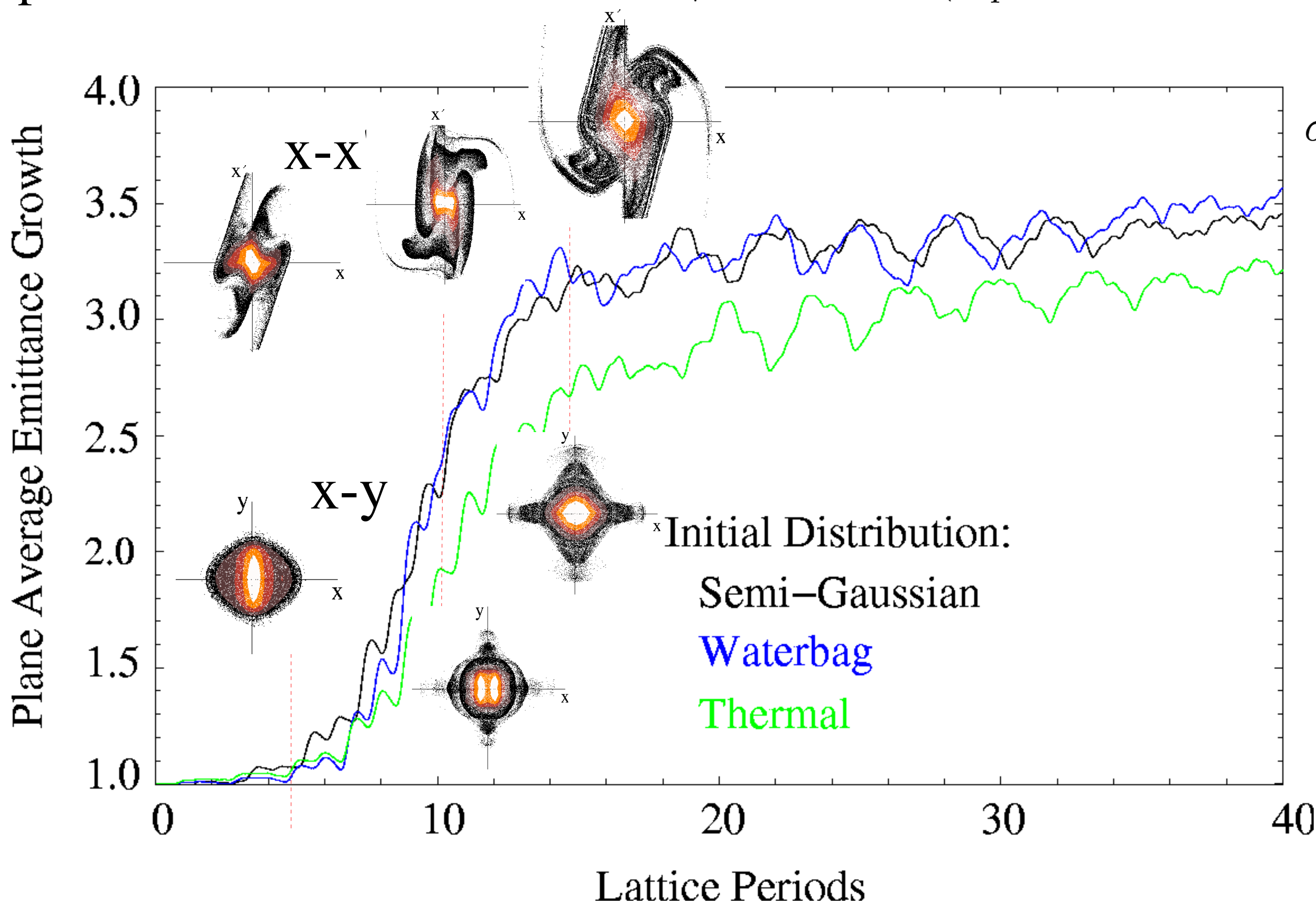
- Density along x- and y-axes for 5 periods
- Emittance growth very small -- 5 period initial transient shown

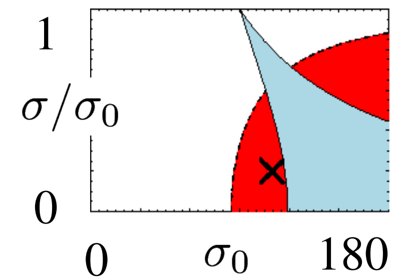


Parametric simulations find broad instability region to the left of the envelope band -- features relatively insensitive to the form of the (non-singular) matched initial distribution

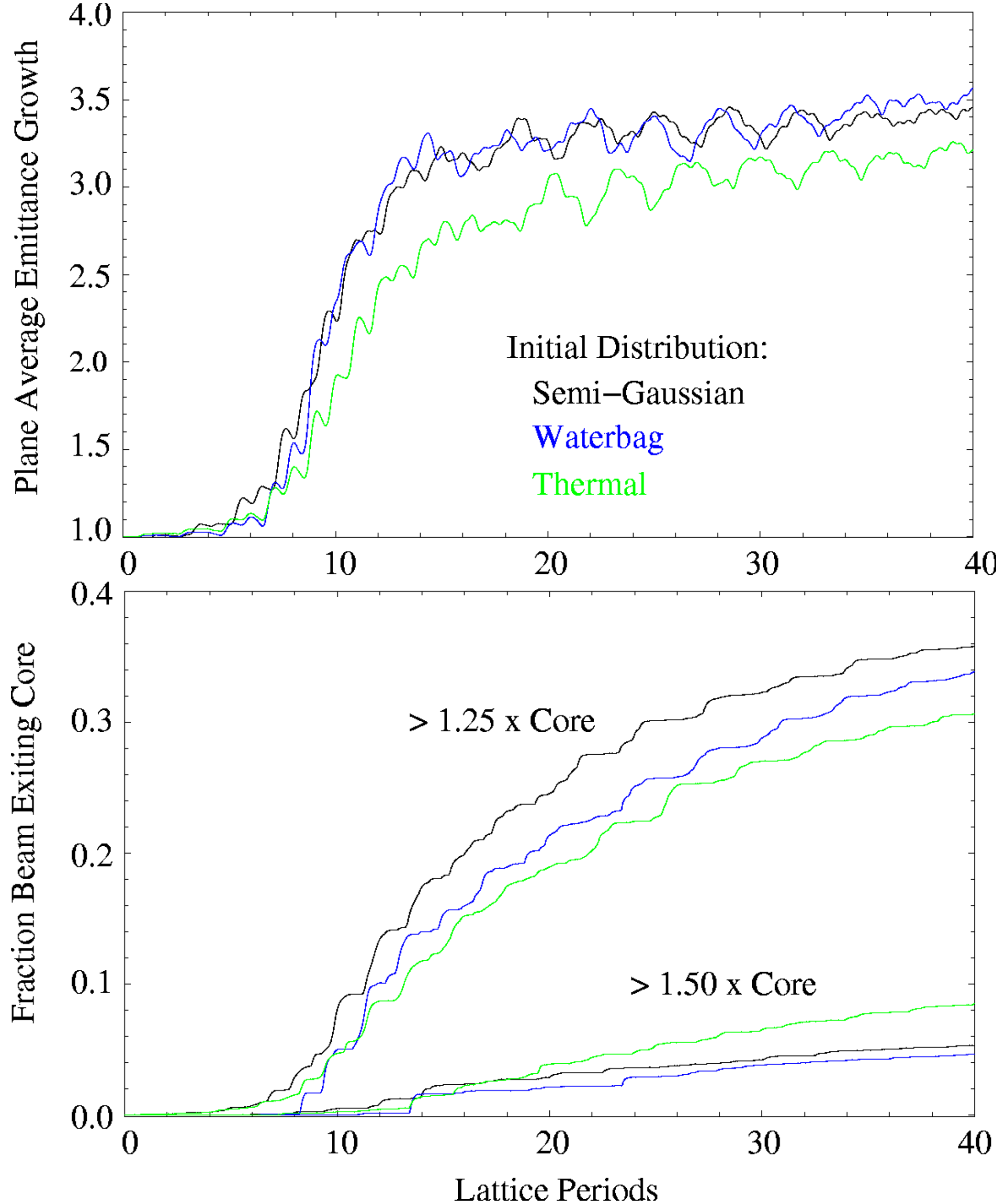
- Where unstable, growth becomes larger and faster with increasing σ_0

Example Parameters: $\sigma_0 = 110°,\ \sigma/\sigma_0 = 0.2\ (L_p = 0.5\ \text{m},\ \eta = 0.5)$





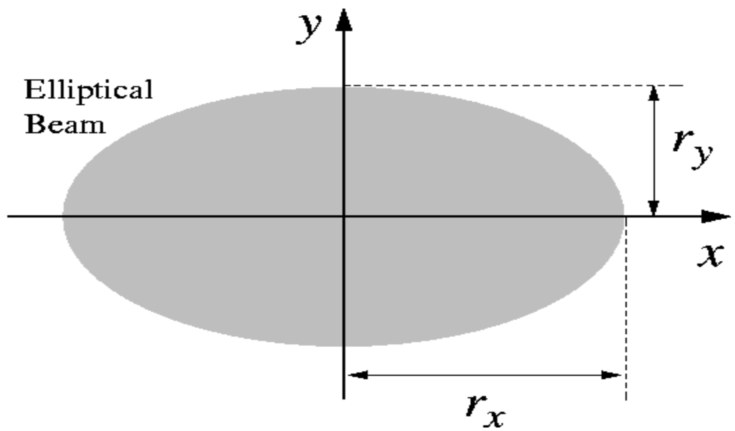
Essential instability feature -- particles evolve outside core of the beam precludes pure "internal mode" description of instability



Instantaneous, rms equivalent measure of beam core:

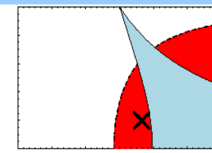
$$r_x = 2\langle x^2\rangle_\perp^{1/2}$$

$$r_y = 2\langle y^2\rangle_\perp^{1/2}$$



"tag" particles that evolve outside core at any *s* in simulation

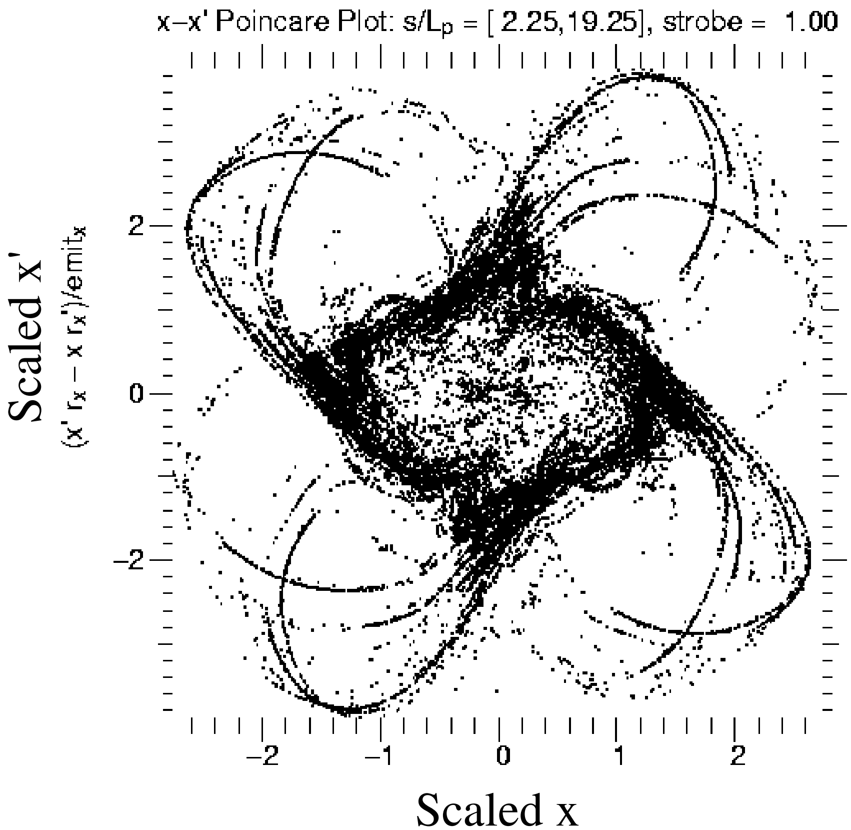
Self-consistent Poincare plots generated show large oscillation amplitude particles have halo-like resonant structure -- qualitative features relatively insensitive to the initial distribution

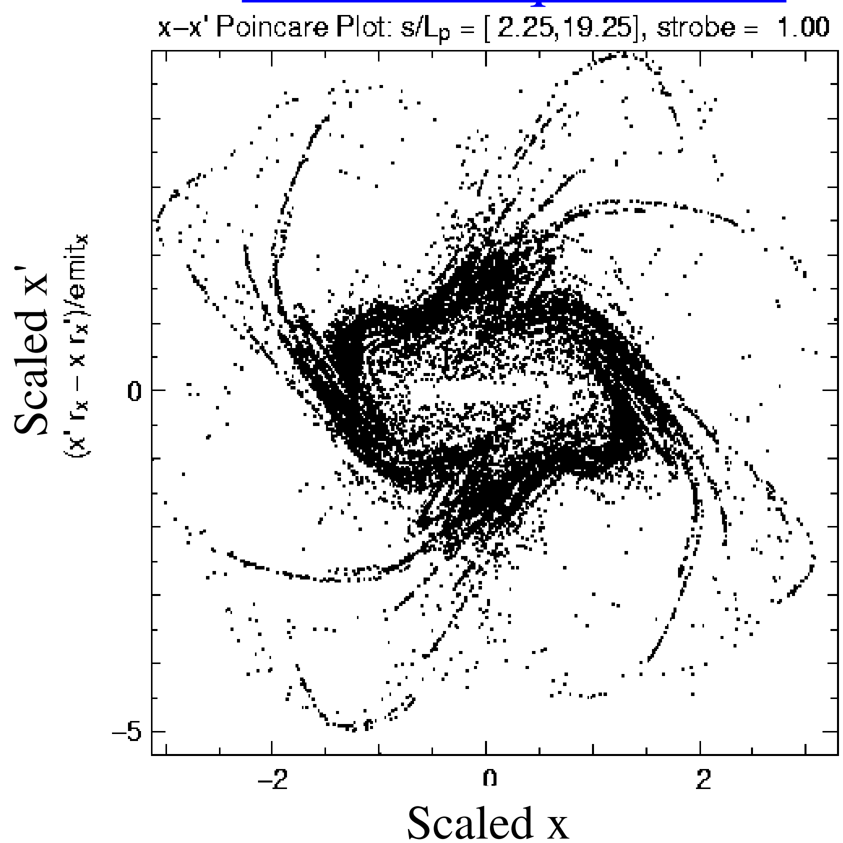


Lattice period Poincare strobe

$$\sigma_0 = 110^\circ \quad \sigma/\sigma_0 = 0.2$$

<u>Semi-Gaussian</u> <u>Thermal Equilibrium</u>





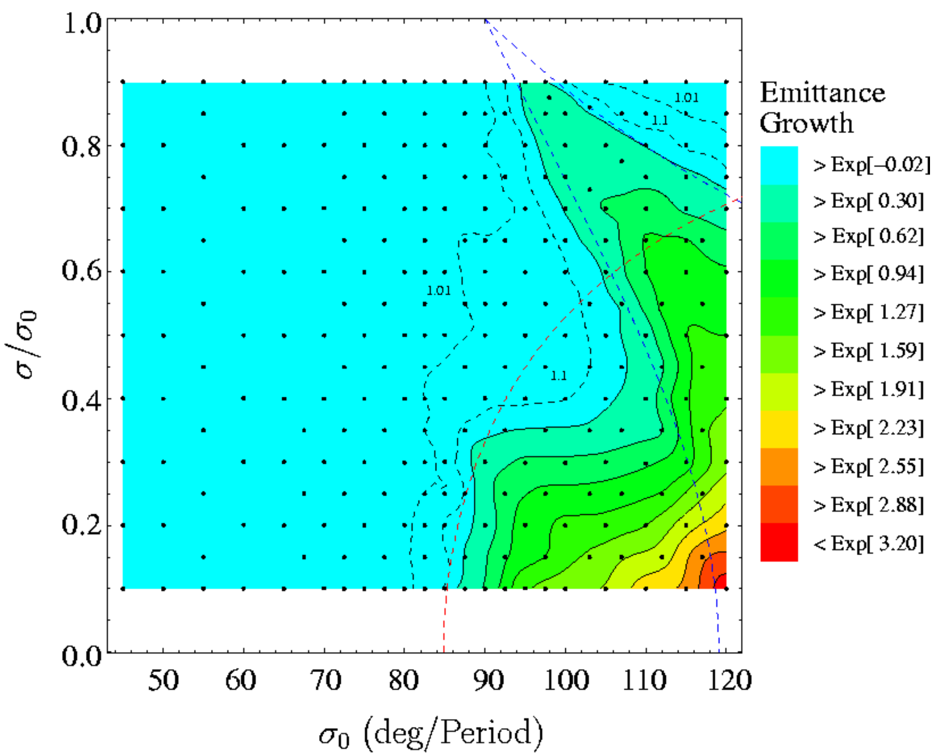
- Particles evolving along *x*-axis particles accumulated to generate clearer picture
  - Including off axis particles does *not* change basic conclusions

Extensive simulations carried out to better understand the parametric region of strong emittance growth

- All simulations advanced 6 undepressed betatron periods
  - Enough to resolve transition boundary: transition growth can be larger if run longer
- Strong growth regions of initial distributions all similar (threshold can vary)
  - Irregular grid contouring with ~200 simulations (dots) thoroughly probe instabilities

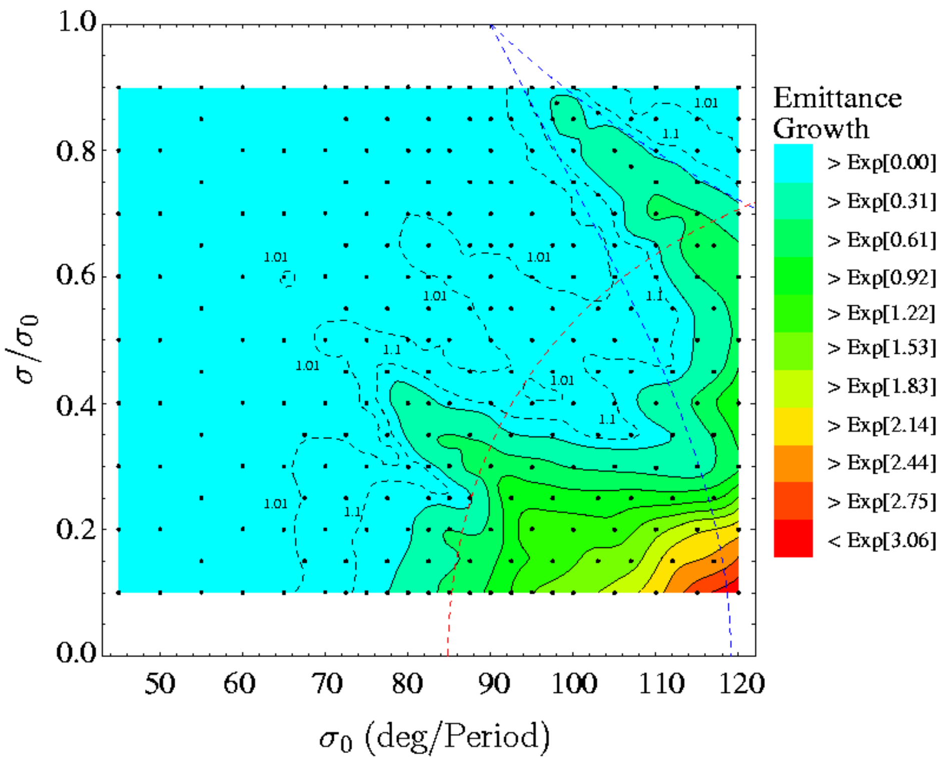
initial semi-Gaussian

- Initial thermal/Gaussian almost identical



initial Waterbag

- Initial KV similar with extra unstable internal modes deep in stable region



Motivated by simulation results -- explore "halo"-like mechanisms to explain observed space-charge induced limits to quadrupole transport

- Resonances can be *strong*: driven by matched envelope flutter and strong space-charge
- *Not* tenuous halo:

  Near edge particles can easily evolve outside core due to:
  - Lack of equilibrium in core
  - Collective waves
  - Focusing errors, ...

  Most particles in beam core oscillate near edge

- Langiel first attempted to apply halo mechanism to space-charge limits

  Langiel, *Nuc. Instr. Meth. A* **345**, 405 (1994)

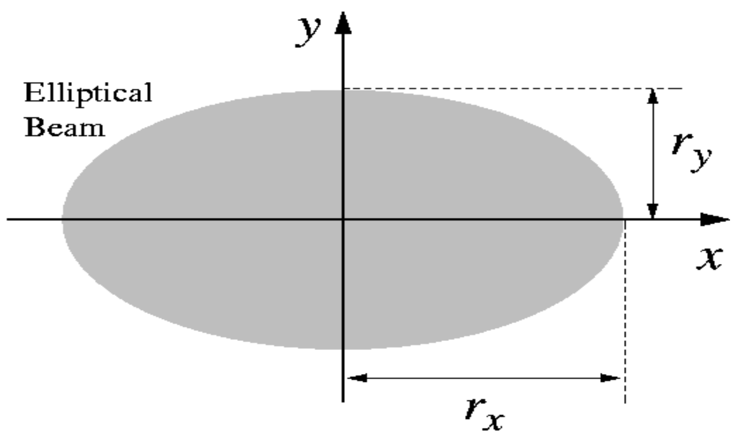
  Appears to concluded overly restrictive stability criterion: $\sigma_0 < 60^\circ$

- Refine analysis: examine halo properties of particles launched just outside the rms equivalent beam core and analyze in variables to reduce "flutter"

  Lund and Chawla, *Nuc. Instr. Meth. A* **561**, 203 (2006)

  Lund, Barnard, Bukh, Chawla, and Chilton, *Nuc. Instr. Meth. A* **577**, 173 (2007)

Core-Particle Model -- Transverse particle equations of motion applied for a test particle moving inside and outside a uniform density elliptical beam envelope



KV envelope equation solved for evolution of beam edge

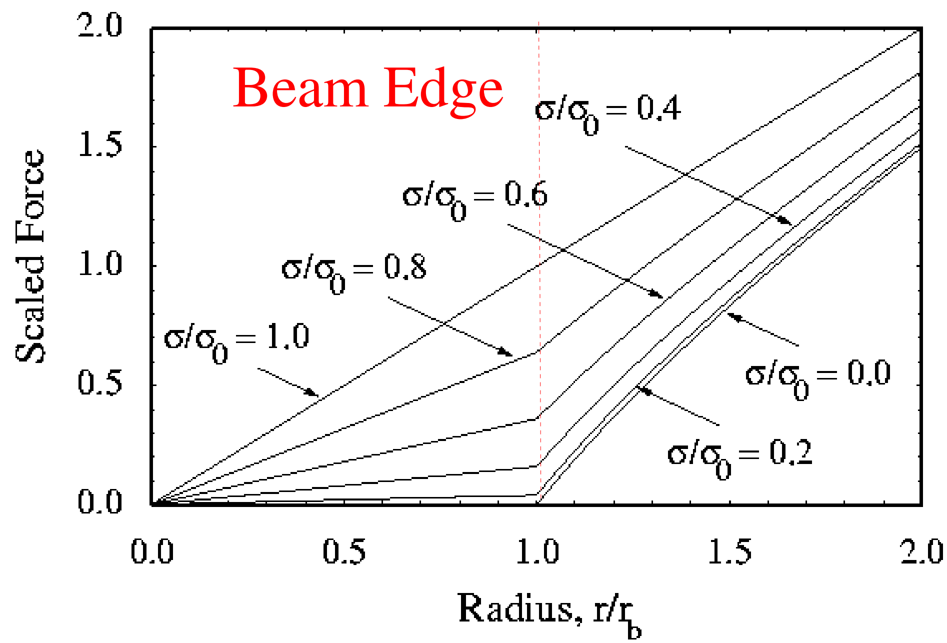
$$r_x'' + \kappa_x r_x - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_x^2}{r_x^3} = 0$$

$$r_y'' + \kappa_y r_y - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_y^2}{r_y^3} = 0$$

$$Q = \frac{q\lambda}{2\pi\epsilon_0 m \gamma_b^2 \beta_b^2 c^2}$$ = dimensionless perveance

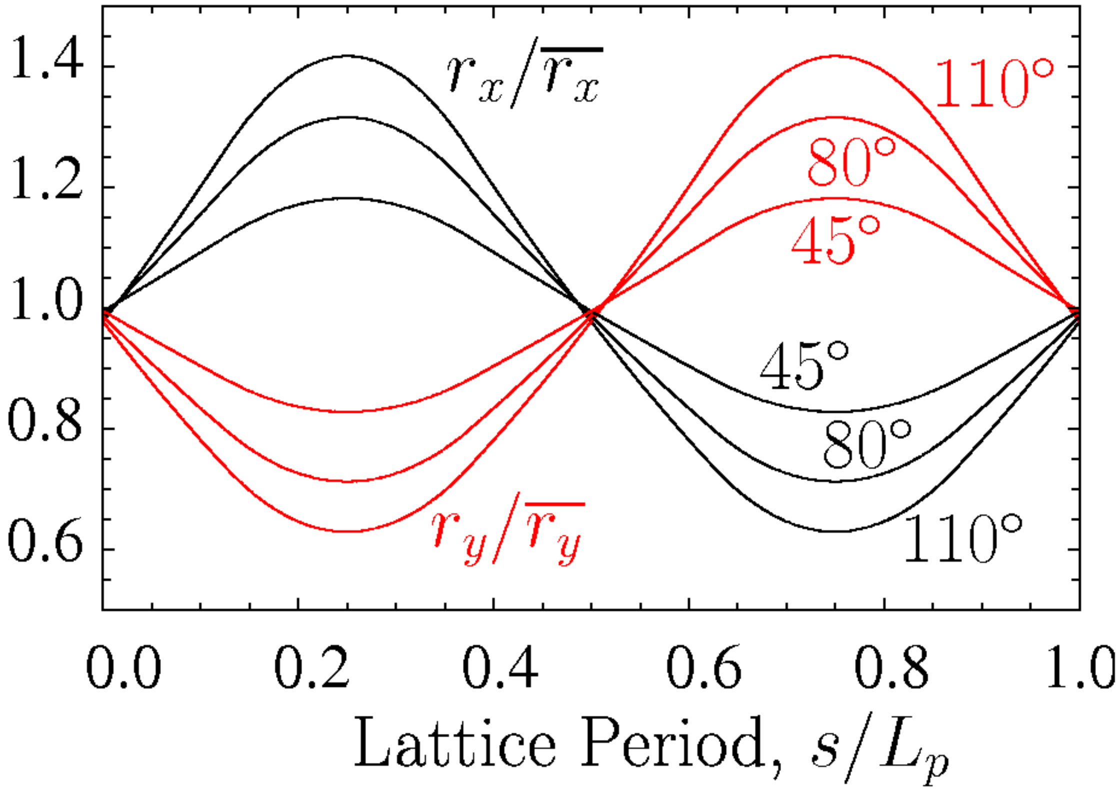
Particles oscillating outside the beam envelope experience amplitude varying nonlinear forces that scale with space-charge intensity

- Nonlinear force transition at beam edge larger for strong space-charge



For quadrupole transport, relative matched beam envelope excursions increase with applied focusing strength

- Larger edge flutter increases nonlinearity acting on particles evolving outside the core



$$\overline{r_x} = \int_0^{L_p} \frac{ds}{L_p} r_x(s)$$

$$\eta = 0.5 \quad L_p = 0.5 \text{ m}$$

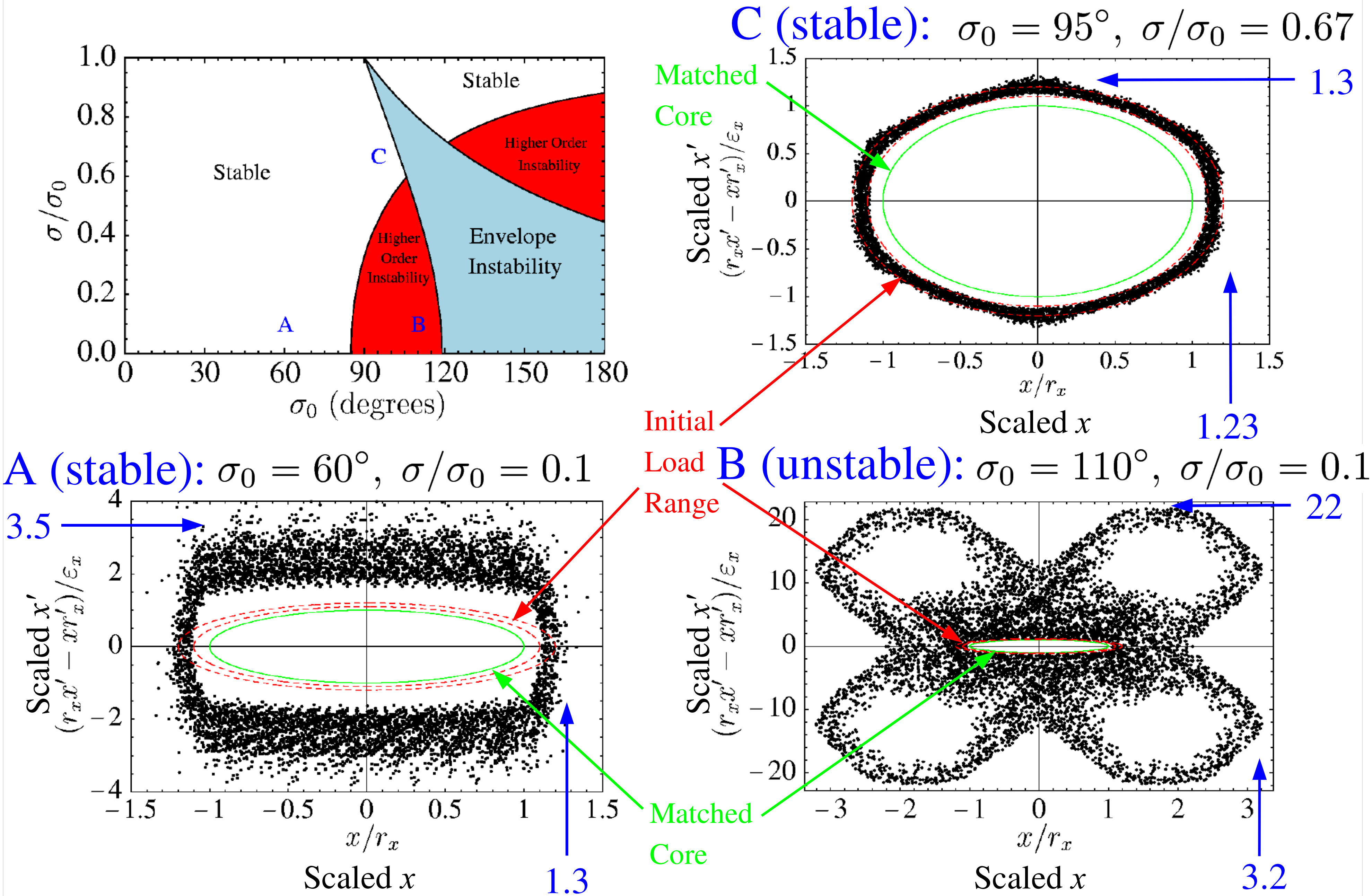
$$Q = 5 \times 10^{-4}$$

$$\varepsilon_x = \varepsilon_y = 50 \text{ mm-mrad}$$

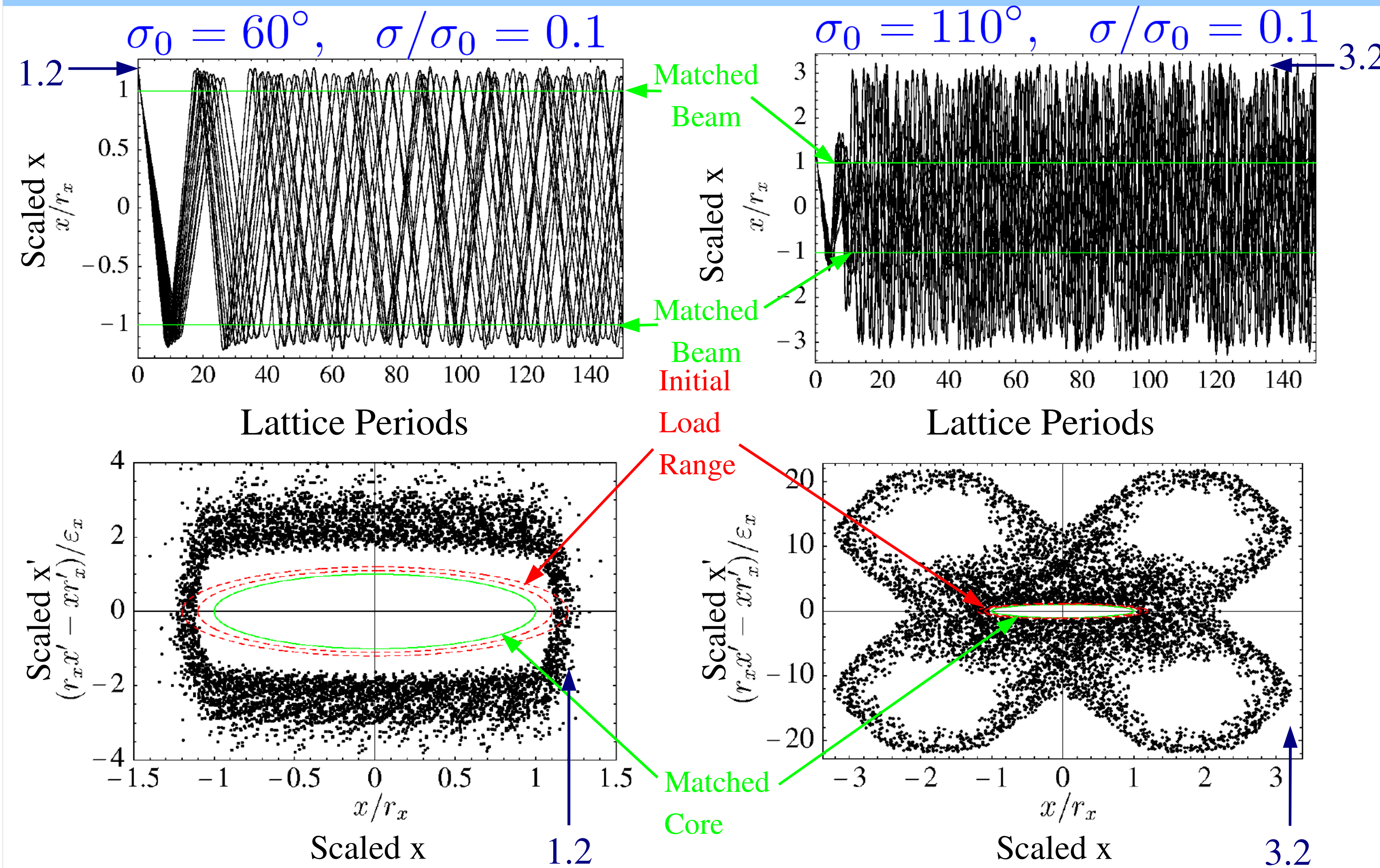
σ_0	σ/σ_0
45°	0.20
80°	0.26
110°	0.32

Space-charge nonlinear forces and *matched* envelope flutter strongly drive resonances for particles evolving outside of beam edge

Core-particle simulations: Poincare phase-space plots illustrate stability regions where near edge particles grow in oscillation amplitude: launch [1.1,1.2]x core



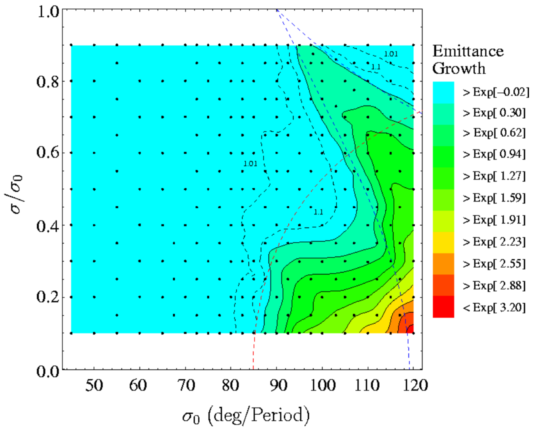
Core-particle simulations: Amplitude pumping of characteristic “unstable” phase-space structures is typically rapid and saturates whereas stable cases experience little or no growth



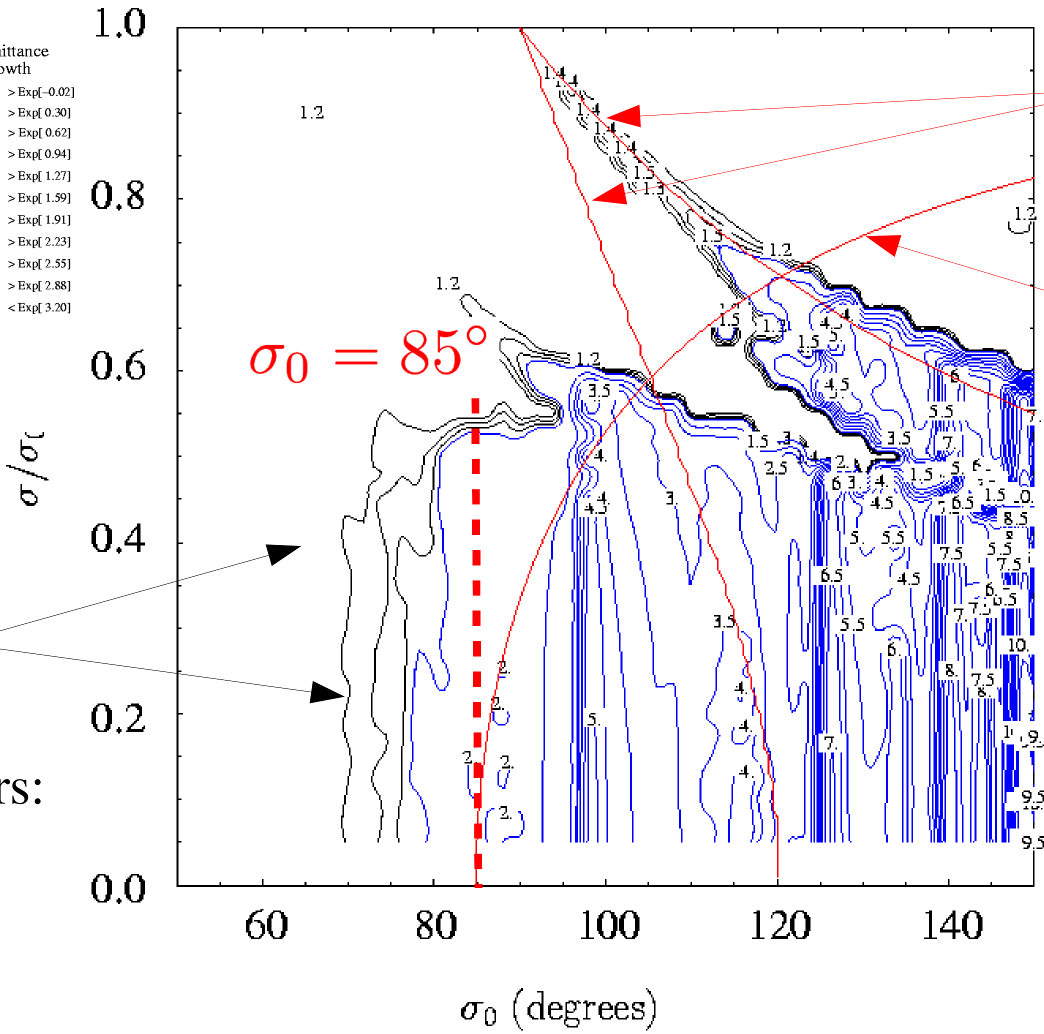
Contours of max particle amplitudes in core particle model suggest stability regions consistent with self-consistent simulations and experiment

Max amplitudes achieved for particles launched [1.05,1.1] times the core radius:

- Variation with small changes in launch position change picture little



PIC Results



Envelope Band Edges

Tiefenback's curve fit to experimental stability boundary

Threshold Growth Black Contours: 1.2, 1.3, 1.4

Large Amplitude Growth Blue Contours: 1.5, 2.0, 2.5, ..., 10

Note: consistent with PIC results, instability well above envelope band not found

Conclusions

High-order space-charge related emittance growth has long been observed in intense beam transport in quadrupole focusing channels with $\sigma_0 \gtrsim 85°$:

- SBTE Experiment at LBNL [M.G. Tiefenback, Ph.D Thesis, UC Berkeley (1986)]
- Simulations by Haber, Laslett, and others

A core-particle model has been developed that suggests these space-charge transport limits result from a strong halo-like mechanism:

- Space-Charge and Envelope Flutter driven
- Results in large oscillation amplitude growth -- strongly chaotic resonance chain which limits at large amplitude rapidly increases oscillations of particles just outside of the beam edge
- Not weak: many particles participate -- Lack of core equilibrium provides pump of significant numbers of particles evolving sufficiently outside the beam edge
- Strong statistical emittance growth and lost particles (with aperture)

Mechanism consistent with other features observed:

- Stronger with envelope mismatch: consistent with mismatched beams more unstable
- Weak for high occupancy solenoid transport: less envelope flutter suppresses

More Details -- Group work on topic:

Overview: simulations and core-particle results

Lund and Chawla, “Space-charge transport limits of ion beams in periodic quadrupole focusing channels,” *Nuc. Instr. Meth. A* **561**, 203 (2006)

Core particle model details: Analysis to periodic quadrupole and solenoid transport

Lund, Barnard, Bukh, Chawla, and Chilton, “*A* core-particle model for periodically focused ion beams with intense space-charge,” *Nuc. Instr. Meth. A* **577**, 173 (2007)

Simulation loads: extensive review including adapted initial beam loads

Lund, Kikuchi, and Davidson, “Generation of intial kinetic distributions for simulation of long-pulse charged particle beams with high space-charge intensity,”
submitted to *PRSTAB*

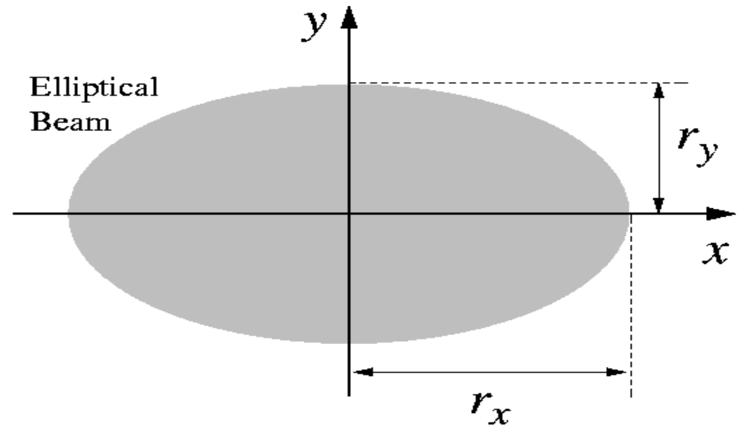
Envelope stability: extensive review

Lund and Bukh, “Stability properties of the transverse envelope equations describing intense ion beam transport,” PRSTAB **7**, 024801 (2004)

Extra Slides

Core-Particle Model -- Transverse particle equations of motion for a test particle moving inside and outside a uniform density elliptical beam envelope

$$\boxed{\begin{aligned} x'' + \kappa_x x &= \frac{2QF_x}{(r_x+r_y)r_x}x \\ y'' + \kappa_y y &= \frac{2QF_y}{(r_x+r_y)r_y}y \end{aligned}}$$



$$Q = \frac{q\lambda}{2\pi\epsilon_0 m\gamma_b^2\beta_b^2 c^2}$$ ..... dimensionless perveance

Where: inside the beam

$$F_x = 1$$

$$F_y = 1$$

outside the beam:

$$F_x = (r_x+r_y)\frac{r_x}{x}Re[\tilde{S}]$$

$$F_y = -(r_x+r_y)\frac{r_y}{y}Im[\tilde{S}]$$

with

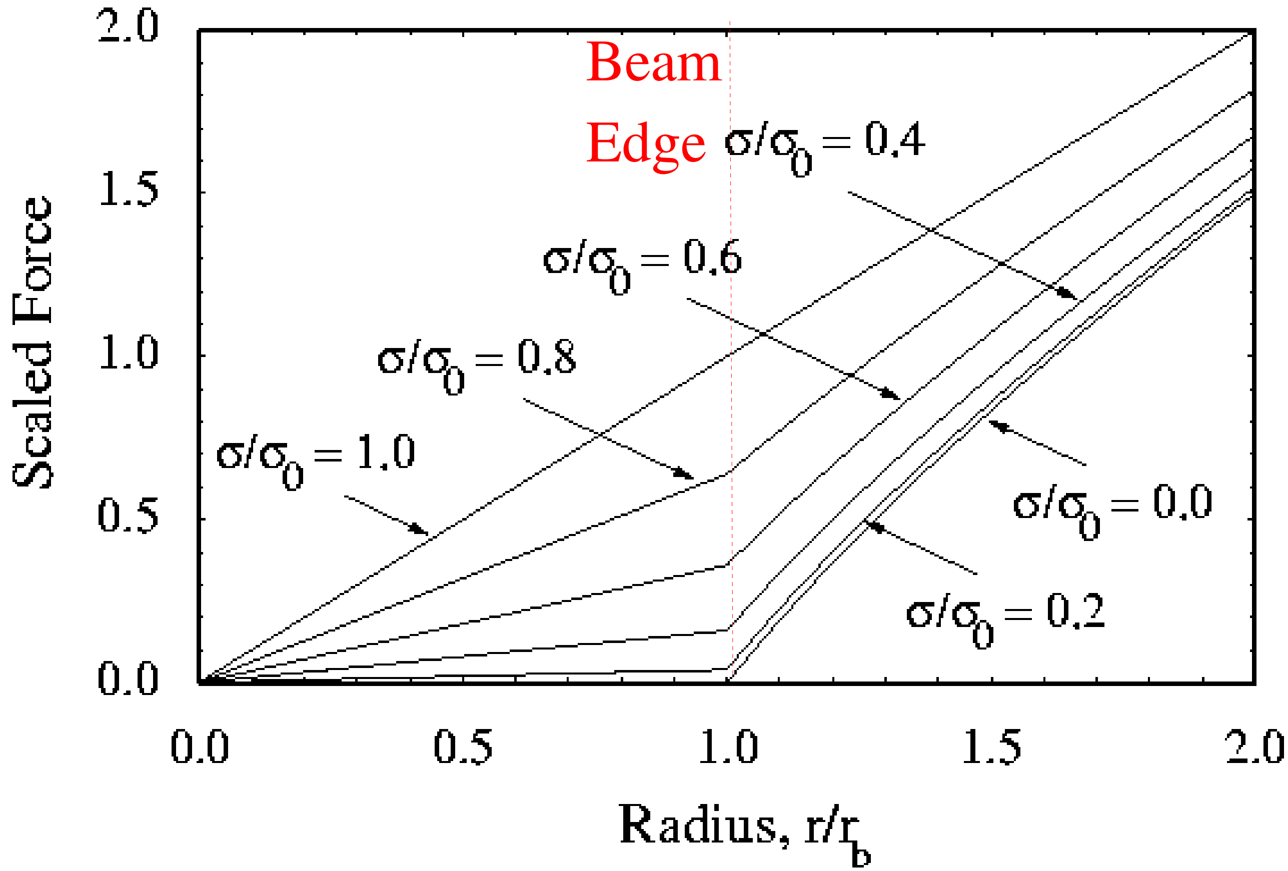
$$\begin{aligned} \tilde{S} &\equiv \frac{\tilde{z}}{r_x^2-r_y^2}[1-\sqrt{1-\frac{(r_x^2-r_y^2)}{\tilde{z}^2}}] \\ &= \frac{1}{2\tilde{z}}\left[1+\frac{1}{2}\frac{r_x^2-r_y^2}{\tilde{z}^2}+\frac{1}{8}\frac{(r_x^2-r_y^2)^2}{\tilde{z}^4}+\cdots\right] \end{aligned}$$

$$\tilde{z} = x + iy$$

$$i = \sqrt{-1}$$

Particles oscillating radially outside the beam envelope experience amplitude varying nonlinear forces that scale with space-charge intensity and can drive strong resonances

Continuous Focusing Axisymmetric Beam Radial Force



- Nonlinear force transition at beam edge larger for strong space-charge

For quadrupole transport, matched beam envelope excursions increase with applied focusing strength -- larger matched edge flutter increases nonlinearity acting on particles evolving outside the core

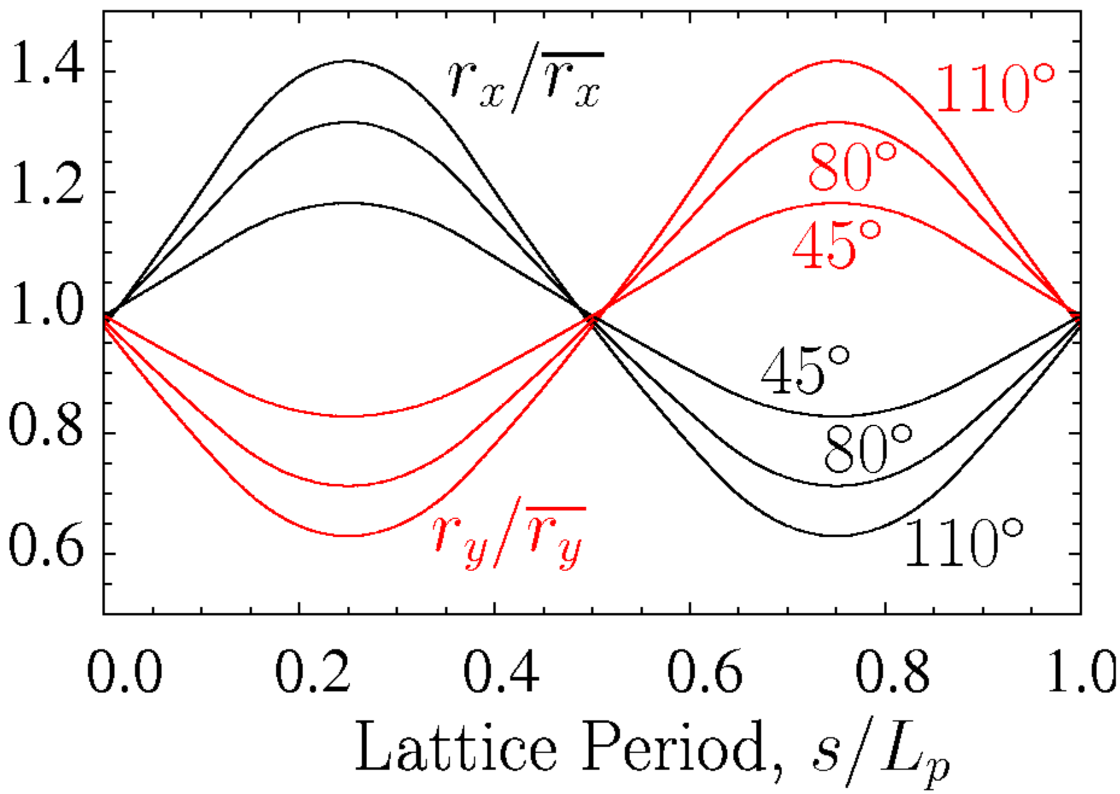
Envelope edge (r_x) flutter scaling for a FODO channel:

$$\overline{r_x} = \int_0^{L_p} \frac{ds}{L_p} r_x(s)$$

$\eta = 0.5 \quad L_p = 0.5$ m

$Q = 5 \times 10^{-4}$

$\varepsilon_x = \varepsilon_y = 50$ mm-mrad



σ_0	σ/σ_0
45°	0.20
80°	0.26
110°	0.32

Core-particle simulations: Poincare plots illustrate resonances associated with higher-order halo production near the beam edge for FODO quadrupole transport

- High order resonances near the core are strongly expressed
- Resonances stronger for higher σ_0 and stronger space-charge
- Can overlap and break-up (strong chaotic transition) allowing particles launched *near the core* to rapidly increase in oscillation amplitude

Lattice Period Poincare Strobe, particles launched [1.1,1.2] times core radius